Calculation of Form Factor for ^{116}Sn and ^{118}Sn by Using Three-Parameter Gaussian Model Density Distribution

Hla Myat Thandar[1], Khin Swe Myint[2]

Abstract

The purpose of research work is to calculate the Form factors of ^{116}Sn and ^{118}Sn nuclei by using three-parameter Gaussian model density distribution. The structural parameters, namely radius parameter (a), skin thickness parameter (z), charge density distribution parameter of ^{116}Sn and ^{118}Sn nuclei, taken from the experimental data are used to get the charge density distribution. After getting the charge density distribution, root mean square radius and the form factor are calculated. At the inner region (<4fm), the density distributions of ^{116}Sn and ^{118}Sn nuclei are slightly different. In the tail part, they are nearly the same for these nuclei. The calculated results of charge density distributions and root mean square radii are in good agreement with the experimental results. Moreover, the form factors of them also agree with the experimental values except 0.5fm^{-1} region.

Keywords: charge density distribution, form factor and root mean square radius

Introduction

There are two types of nuclear distributions which are nuclear charge distribution and nuclear matter distribution. The charge distribution is mainly determined by the arrangement of the protons and the matter distribution by those of protons and neutrons combined. Our calculation is associated with charge density distribution. In theory, charge density distribution for various form factors depend on nuclear model. At the present work, the charge density distribution will be calculated with three-parameter Gaussian model charge density distribution. A compilation of nuclear charge density distribution parameters is obtained from elastic electron scattering experiment. The electrons give us information on charge distribution and neutrons give us information on matter distribution. In the nucleus, there are two main forces that we have to consider: the electromagnetic force and nuclear force. The former interests particles with an electric charge (protons, electrons), the latter holds protons and neutrons together. The nuclear force is not a fundamental force but a remnant of the strong interaction (which binds quarks to form neutrons and protons) like the Vander Waals forces that allow chemical bonds between molecules and remnants of the electromagnetic force within each atom and molecule.

If we also want to know how neutrons are distributed then we should bombard the nucleus with particles that respond to nuclear force only and the nuclear choice is other neutrons. As far as we know, electron scattering has a higher resolution and therefore provides more detailed information than neutron scattering. The fact is that the neutrons are actually composed of quarks and each of them might be interacting with the quarks of protons and neutrons in the nucleus.

The neutron scattering yields worse results than electron scattering but according to the description, they are compatible with one another. So, we get the distribution of protons (assuming charge distribution) from electron scattering and assume that the distribution of neutrons (assuming is matter distribution) is the same [1].

[1] Lecturer, Dr., Department of Physics, University of Mandalay

[2] Emeritus Professor, Dr, Department of Physics, Mandalay University

Nuclear Charge Distribution

The early theorists, without access to computers, had strong reasons to use analytical descriptions of charge density distributions and potentials, that enabled series expansions of analytical solutions of the wave functions within and close to the nucleus. A common choice was the homogenous charge distribution inside a radius $R = R_0 A^{1/3}$, where A is the mass number of nucleus. The most important parameter for many properties is the expectation value $<r^2>$ which has the value $3R^2/5$for the homogenous nucleus. Already this simple distribution gives the correct analytical behavior. These expansions are also useful for general understanding of the effects involved [2].

Nuclear Electric Form Factor

In an electron elastic scattering process with a target nucleus, we have non relativistic differential cross section (Rutherford) and relativistic differential cross section (Mott). The differential cross section of Rutherford scattering does not include the spins while Mott scattering takes into account the spin of the colliding particles. In both cases, it is assumed that the target nucleus is a point charge. For a target having an extended finite size, the differential cross section differs from that of the Rutherford or Mott by a factor called form

factor F (q) such as

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}_{Mott(or)Rutherford} \left|F(q)^2\right|. \qquad (1)$$

Thus, the form factor indicates the effect of the nuclear size upon the differential cross section, where q is the momentum transfer of the scattering process. The form is known as the Fourier transform of the charge density distribution,

$$F(q) = \frac{1}{Z}\int \rho_{ch}(r)\, e^{i\vec{q}.\vec{r}}\, d\vec{r} \qquad (2)$$

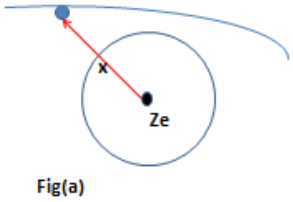
and the verification of the above relation will be given later. The form factor plays an important role because it is the most important link between experimental observation and theoretical analysis. The form factor is the direct result of a cross section measurement. From the theoretical side, charge density distribution $\rho_{ch}(r)$ is a solution of Schrodinger equation.

Experiment Comparison Theory

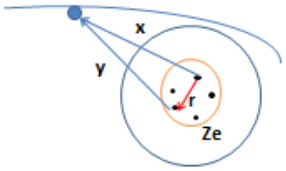
$$\frac{d\sigma}{d\Omega} \rightarrow \left| F(q^2) \right| \Leftrightarrow F(q^2) \leftarrow \rho(r) \leftarrow \psi(r) \leftarrow \text{Schrodinger equation}$$

Derivation of The Relation between Form Factor and Density Distribution

Let us compute the scattering an electron by a spherically symmetric nucleus having a finite size of Figure.



Fig(a)



Fig(b)

Figure(a) shows scattering of a point electron by a point charge nucleus

Figure (b) scattering of a point electron by a nucleus with extended charge distribution.

Figure (1) Scattering of a point electron by (a) a point charge nucleus (b) a nucleus with extended charge distribution

The screened Coulomb potential between an electron and a point charge nucleus

$$V(x) = -(Ze^2/x)e^{x/a} \quad (1)$$

An infinitesimal volume element drcontains a charge $dq = Ze\rho_{ch}(r)dr$ which gives a contribution

$$dV(x) = \frac{-Ze^2}{y} e^{\frac{-y}{a}} \rho_{ch}(r)d\vec{r}\,, \text{ where } \vec{y} = \vec{x} - \vec{r} \quad (2)$$

$$V(x) = -Ze^2 \int \frac{e^{\frac{-y}{a}}}{y} \rho_{ch}(r)d\vec{r} \quad (3)$$

The first Born approximation for a scattering amplitude f (q) is,

$$f(q) = -\frac{m}{2\pi\hbar^2} \int V(x) e^{i\vec{q}.\vec{x}} d\vec{x} \quad (4)$$

where $q = 2k\ sin\frac{\theta}{2}$ is the momentum transfer of the elastic scattering process with scattering angle θ.

By substituting the V(x) in equation (4), we obtain

$$f(q) = \frac{mZe^2}{2\pi\hbar^2} \int d\vec{r}\, e^{i\vec{q}.\vec{x}} \rho_{ch}(r) \int d\vec{y} \frac{e^{\frac{-y}{a}}}{y} e^{i\vec{q}.\vec{y}}\,. \quad (5)$$

After solving the equation (5), scattering amplitude f (q) can be written as

$$f(q) = \frac{mZe^2}{2\pi\hbar^2} \int dr\, e^{i\vec{q}.\vec{r}} \rho_{ch}(r) \frac{4\pi}{q^2}\,. \quad (6)$$

Since, $\frac{d\sigma}{d\Omega} = |f(q)|^2$ by squaring both sides of equation (6)

$$\frac{d\sigma}{d\Omega} = |f(q)|^2 = \frac{4m^2Z^2e^4}{\hbar^4 q^4} \left| \int dr\, e^{i\vec{q}.\vec{r}} \rho_{ch}(r) \right|^2. \quad (7)$$

However, the differential scattering cross section for electron scattering by a point charge nucleus (Rutherford scattering) is known to be

$$(\frac{d\sigma}{d\Omega})_{Rutherford} = \frac{4m^2Z^2e^4}{\hbar^4q^4}|F(q)|^2. \quad (8)$$

By comparing equation (7) and (8),

$$(\frac{d\sigma}{d\Omega}) = (\frac{d\sigma}{d\Omega})_{Rutherford}|F(q)|^2$$

where, the form factor is .

$$F(q) = \int dr\, e^{i\vec{q}.\vec{r}}\rho_{ch}(r). \quad (9)$$

After solving the equation (9), the relation between Form Factor and Charge Density Distribution [3] is as follows:

$$F(q) = \frac{4\pi}{Z}\int_{r=0}^{\infty} r\,\rho_{ch}(r)\quad \frac{\sin qr}{q}dr. \quad (10)$$

Three-Parameter Gaussian Model

In our calculation, three-parameter Gaussian model is used to calculate the root mean square radius, charge density distribution and form factor.

Using a Gaussian charge distribution then has the advantages of making it possible to evaluate electron-nucleus interactions using the same integral routines as the electron-electron interaction. An advantage of the use of Gaussian is that values of $\rho(r)$ at different values of r are decoupled to a large extent because of the rapid decrease of the Gaussian tail. The results of the analysis are independent of the number of Gaussians, provided this number is sufficiently large to allow a good fit to data [2].
For three-parameter Gaussian model, charge density distribution is

$$\rho(r) = \frac{\rho_0(1+\frac{wr^2}{c^2})}{(1+e^{(r^2-c^2)/z^2})}$$

where, c= radius parameter (or) diffuseness parameter

z= the skin thickness parameter

w= the charge density distribution parameter

The parameters of three - parameter Gaussian model are expressed in Table (1).

Table (1) Charge density distribution parameters for three-parameter Gaussian model [4]

No.	Nucleus	c (or) a (fm)	z (or) α (fm)	w
1.	^{116}Sn	5.062	2.625	0.272
2.	^{118}Sn	5.058	2.625	0.295

Calculation of RMS and Charge Density Distribution for Three Parameter Gaussian Model

Density distribution for three-parameter Gaussian model is

$$\rho(r)=\frac{\rho_0(1+\frac{wr^2}{c^2})}{(1+e^{(r^2-c^2)/z^2})} \tag{1}$$

The normalization condition for the density distribution is

$$\int \rho(r)\, dV = Ze$$

$$\int \rho(r)\ r^2 dr = \frac{Ze}{4\pi} \tag{2}$$

By substituting equation (1) in equation (2),

$$\int \frac{\rho_0(1+\frac{wr^2}{c^2})}{(1+e^{(r^2-c^2)/z^2})} r^2 dr = \frac{Ze}{4\pi}$$

$$\rho_0 = \frac{1}{\int \frac{(1+\frac{wr^2}{c^2})}{(1+e^{(r^2-c^2)/z^2})} r^2 dr} \frac{Ze}{4\pi} \tag{3}$$

Root mean square radius is calculated as follows:

$$\left\langle r^2 \right\rangle^{1/2} = \int \psi^* r^2 \psi dV$$

$$\left\langle r^2 \right\rangle^{1/2} = \frac{4\pi}{Ze} \int \rho(r) r^4 dr \tag{4}$$

Results and Discussion

Root mean square radius and charge density distribution of ^{116}Sn and ^{118}Sn nuclei are determined to obtain the Form Factors. Three parameter Gaussian model charge density distributions are used for these nuclei. Three-parameters are that c or a is radius parameter, z or α is skin thickens parameter and w is charge density distribution parameter for these nuclei. The tail regions of charge density distribution for these nuclei are nearly the same, the top regions are not. It may be due to the effect of the number of neutrons. These results are expressed in Figure (2).

The calculated and experimental values of the root mean square radius are expressed in Table (2). It can be seen that the calculated values are in good agreement with the experimental values. The charge density distributions of them are in good agreement between experimental and calculated results which are expressed in Figure (2) to Figure (4). Then, the form factors of these nuclei agree with the experimental values except 0.5fm^{-1} region. They are shown in Figure (5) and Figure (6).

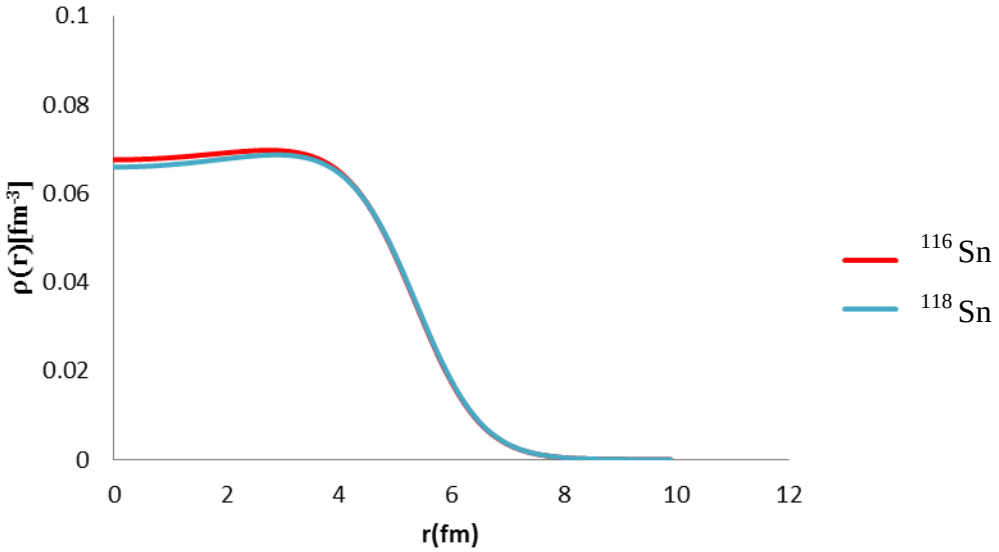


Figure (2) Charge Density Distribution of ^{116} Sn and ^{118} Sn nuclei in 3PG models

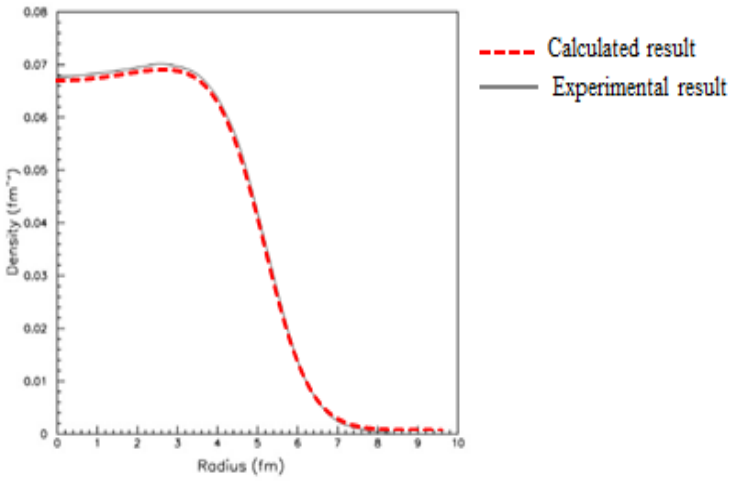


Figure (3) Calculated and experimental results [5]of charge density distribution for ^{116}Sn

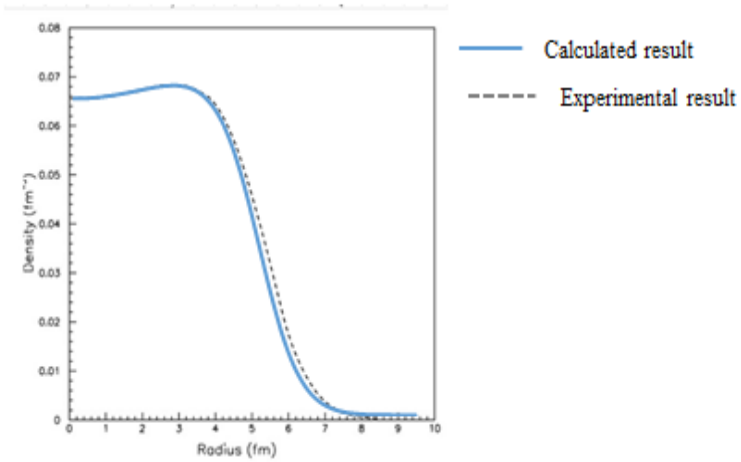


Figure (4) Calculated and Experimental results [5] of charge density distribution for ^{118}Sn

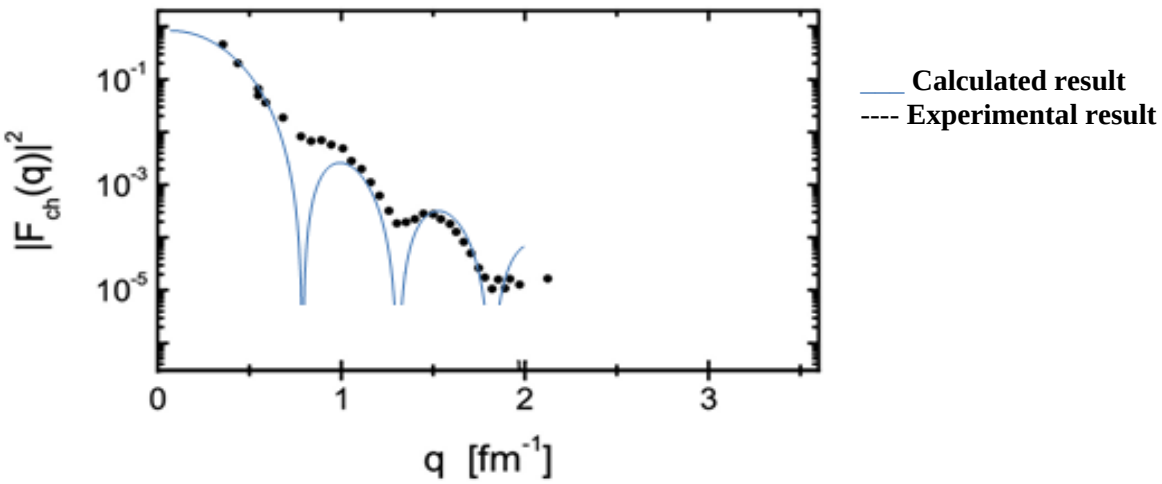


Figure (5) Calculated and Experimental results [6] of form factor for ^{116}Sn

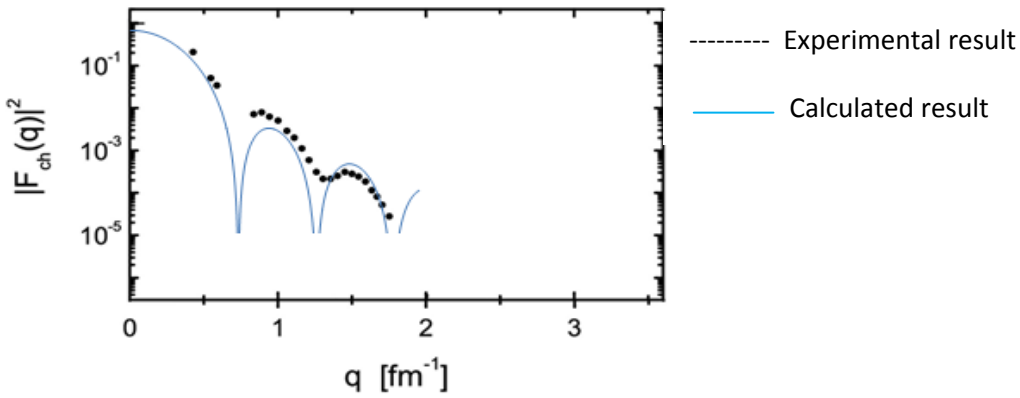


Figure (6) Calculated and Experimental results [6] of form factor for ^{118}Sn

Table (2) Comparison between our calculated results and experimental results for RMS values of ^{116}Sn and ^{118}Sn in 3PG model

No.	Nucleus	Our calculated results (fm)	Experimental results (fm)
1.	^{116}Sn	4.618	4.619
2.	^{118}Sn	4.634	4.634

Acknowledgement

The authors would like to acknowledge to Professor Dr Lei Lei Win, Head of Department of Physics, University of Mandalay, for her permission. The authors also thank to Pro-Rectors, University of Mandalay, for their encouragement.

References

[1] https://Physics.stackexchange.com

[2] E.R. Schneuwly, H. Vuilleumier, J.L. Walter, *et.al*, "Atomic Data and Nuclear Data Table" (1974)

[3] E.M. Henaly, "Subatomic Physics", Second edition, (1991)

[4] H. Vries, C.W. Jager and C. Vries, "Atomic Data and Nuclear Data Table" (1987)

[5] J. Bartel, P. Quentin and M. Brack, *et al*., Phys. Rev. A**79** (1982)

[6] A.N. Antonov, *et.al*, Phys. Rev. C**72** (2005)